Derivatives

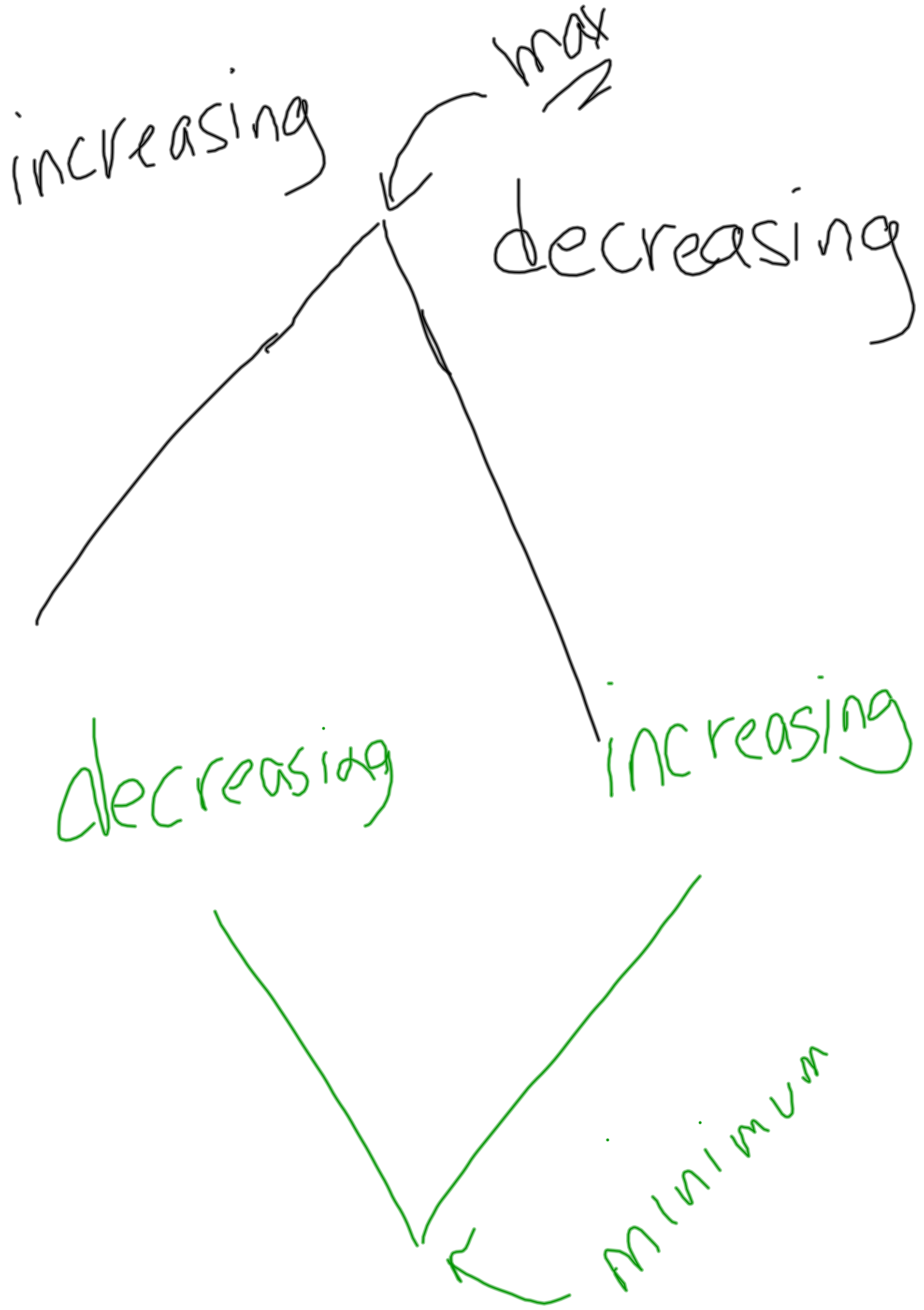
rules
- power
- product
- quotient
- chain

def
limit . . .

- instantaneous rate of change
- at any x-value is the slope of the tangent line at that x-value
- (5.1) can be used to determine increasing / decreasing nature of original function
- differentiability $\Rightarrow$ continuity

5.2 Basic Idea

Continuous



A critical point is a point on the curve at which $f'(x)=0$ [there is a horizontal tangent] $\boxed{\text{OR}}$ $f'(x)$ is not defined.

[reason: maximum/minimum values of a <u>continuous</u> function can only occur at critical points]

[WARNING: critical points can exist without extrema]

$y'=3x^2=0$
$\Rightarrow x=0$

A <u>critical number</u> is the x-value of a critical point.

$f'(x)$ → determine x's where f' is undefined

$f'(x)$ → determine x's where $f'=0$

5.2 7a) $f(x) = x^3 + 3x^2 - 9x + 1$

1) $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3)$

$\rightarrow$ always defined [[no undefined critical numbers]]

$\hookrightarrow$ i.e. no crit ~~#s~~ that result from f' undefined

$(x+a)(x+b)$
$= x^2 + ax + bx + ab$

$\rightarrow f'(x) = 0$

$3(x^2 + 2x - 3) = 0$

$3(x\ {}^{+1})(x\ {}^{-3}) = 0$

$-1 \quad +3$

$3(x-1)(x+3) = 0$

$\hookrightarrow x+3 = 0 \Rightarrow x = -3$

$\hookrightarrow x-1 = 0 \Rightarrow x = +1$

$\Big\}$ critical numbers

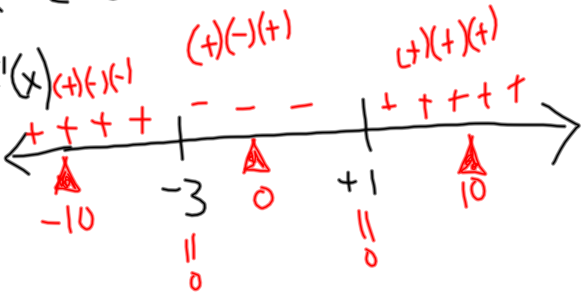
$f(x) = x^3 + 3x^2 - 9x + 1$

1) $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x-1)(x+3)$

critical numbers = +1, -3

2) STRATEGY: check each critical #

2a) sign chart of $f'(x)$



2b) at $\boxed{x=-3}$

$f'(x)$ changes from + to −

so I have a relative maximum

local maximum

at $\boxed{x=+1}$

$f'(x)$ changes from − to +

so I have a relative minimum

local minimum

Always works

$f(x) = x^3 + 3x^2 - 9x + 1$

1) $f'(x) = 3x^2 + 6x - 9 = 3(x^2 + 2x - 3) = 3(x-1)(x+3)$

critical numbers $= +1, -3$

2alternative STRATEGY: to check each critical number

$\rightarrow f''(x) = 6x + 6 = 6(x+1)$

$\rightarrow$ at $x = -3$

evaluate $f''(-3) = 6(-3+1) = -12$

$-12 < 0$ so $f(x)$ is concave down

$f(-3)$ has a horizontal tangent [[b/c $f'(-3) = 0$]]

[[ ]] so $f(x)$ has a rel max at $x = -3$

$\rightarrow$ at $x = +1$

$f''(+1) = 6(1+1) = +12 > 0$. $f(x)$ is concave UP

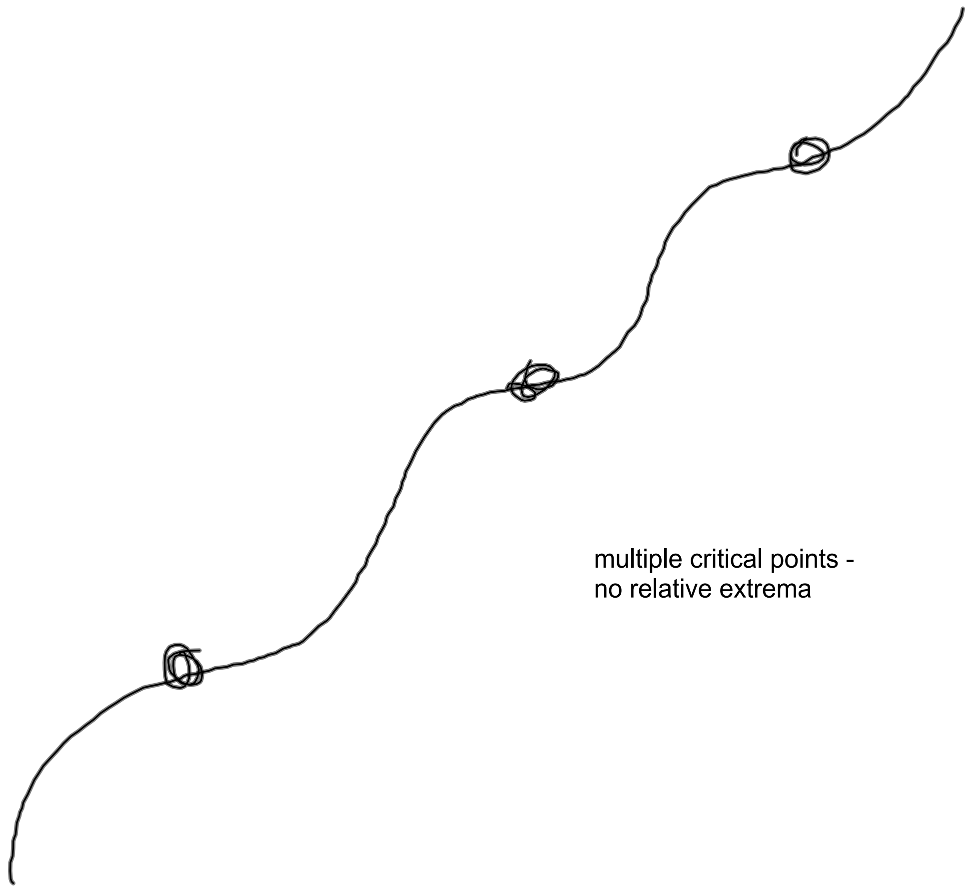
so $f(x)$ has a rel. min. at $x = +1$

[[ ]] so $f(x)$

"Quicker" once you have f''

But if $f''(x) = 0$ IDK anything

and f'' might not exist



multiple critical points -
no relative extrema

Nov 9-8:23 AM